J. Fluid Mech. (1976), *vol.* 78, *part* 3, *pp.* 601–607
Printed in Great Britain

Sizes and shapes of salt fingers near the marginal state

By C. F. CHEN AND R. D. SANDFORD

Department of Mechanical, Industrial and Aerospace Engineering,
Rutgers University, New Brunswick, New Jersey 08903

(Received 19 January 1976 and in revised form 21 June 1976)

Experiments were carried out to investigate the onset, size and shape of fingers in a salt–sugar two-component system. The results on the critical Rayleigh number ratio for the marginal state agree well with the prediction of the linear stability theory of Stern (1960). The theory also predicts that the fingers at the marginal state should be as wide as the layer thickness. In our experiments, in a layer of approximately 3·5 cm deep, the fingers, when they appeared, were always long and narrow, with typical lateral dimensions of approximately 1 mm.

---

1. Introduction

When a layer of warm salt water is above a layer of comparatively colder and fresher water, finger instability can occur at the interface even though the entire system is gravitationally stable. This is a manifestation of double-diffusive instability, which is caused by the difference in the molecular diffusivities of salt and heat. Such an instability was first conceived by Stern (1960), on the basis of an 'oceanographic curiosity' described by Stommel, Arons & Blanchard (1956). In Stern's paper, a linear perturbation analysis was made of a layer of fluid with constant positive upward gradients of salt and temperature. The fluid was assumed to have constant properties. In the determination of the marginal state, the problem reduced to an equivalent Bénard problem. For free–free boundary conditions, the critical condition is

$$R_S = \tau(R_T + \tfrac{27}{4}\pi^4), \tag{1}$$

where the S-Rayleigh number $R_S = g\beta(\partial S/\partial z)\,h^4/\nu\kappa_T$ and the T-Rayleigh number $R_T = g\alpha(\partial T/\partial z)\,h^4/\nu\kappa_T$. Subscripts S and T denote salt and temperature (or the slower and faster diffusing solutes) respectively, κ_S and κ_T are their respective molecular diffusivities with $\tau = \kappa_S/\kappa_T$, g is the gravitational acceleration, ν is the kinematic viscosity, h is the depth of the layer and z is the vertical co-ordinate. The volume expansion coefficients are

$$\alpha = -\frac{1}{\rho}\frac{\partial\rho}{\partial T}, \quad \beta = \frac{1}{\rho}\frac{\partial\rho}{\partial S}.$$

The density of the mixture is approximated by

$$\rho = \rho_0[1-\alpha(T-T_0)+\beta(S-S_0)],$$

where the subscript zero denotes the reference state. The fingers or convection cells, according to the linear theory, should have a horizontal dimension approximately the same as the depth h. In the same paper, Stern reported an experiment done by Stommel & Faller using salt and heat. The fingers appeared as long narrow cells. Considering a linear analysis at the experimentally appropriate supercritical conditions, Stern found that the fastest growing wave had a wavelength $l/h \sim R_T^{-\frac{1}{4}}$. Nield (1967)† has extended the stability analysis to more general boundary conditions. The results show that the constant term in the critical condition (1) and the critical wavelengths remain of the same order of magnitude.

Stern & Turner (1969) noted that the same phenomenon would occur if the fluid consisted of two solutes with different molecular diffusivities. Shirtcliffe & Turner (1970) obtained some striking shadowgraphs showing the planforms of the fingers which existed between a layer of sugar solution above a layer of salt solution. Huppert & Manins (1973) determined that, for the two-layer system, if $\alpha\Delta T/\beta\Delta S < \tau^{-\frac{3}{2}}$ finger instability will occur. They conclude that this condition "should be satisfied in all parts of the ocean where hot, salty water overlies colder, fresher water". Linden (1973) investigated the structure of fingers in a sugar–heat system. It has been suggested by Turner (1967) that salt fingers may be responsible for the layering structure observed underneath the Mediterranean outflow. This conjecture was confirmed by a recent *in situ* observation made by Williams (1974) in the North Atlantic west of Gibraltar, at a depth of approximately 1300 m.

In all these experiments cited, whether done in the laboratory or in the ocean, the salt fingers were always long and slender. The values of R_S in these experiments, except the series by Huppert & Manins, were all in the supercritical range. It is consistent with the theoretical considerations that the cells be long and narrow. In the case of Huppert & Manins, the conditions were close to the marginal state. However, the length scale h must be the depth of the diffusion layer, which was rather small at the onset of instability. It may be argued that the cells were initially as wide as they were tall, but, as the diffusive layer deepened, the cells locked into their original horizontal dimensions and again appear as slender fingers. In view of these results, the following question presents itself: is the long slender cell a peculiar phenomenon which only occurs at supercritical conditions or is it the shape the convection cells assume even at marginal states?

One way to answer this question is to perform experiments such that (i) the values of R_S and R_T are so large that the constant term in (1) may be neglected, and (ii) the ratio $R_S/R_T = \lambda$ is constant throughout the layer prior to the onset of fingers. The value of λ is to be varied to determine the critical condition for the appearance of salt fingers. Some means of visualizing the fingers should be provided to determine their sizes and shapes. One method which comes to mind immediately is to fill a test tank in the manner of Turner & Chen (1974) to obtain a layer of fluid with continuous but opposing gradients of salt and sugar. We indeed carried out preliminary tests in this manner. It was noticed that, for an initial value of $\lambda > 0{\cdot}25$, fingers always appeared first in the regions near the top and bottom boundaries. They then grew slowly towards each other. The fingers

---

† We thank H. Huppert for calling our attention to this reference.

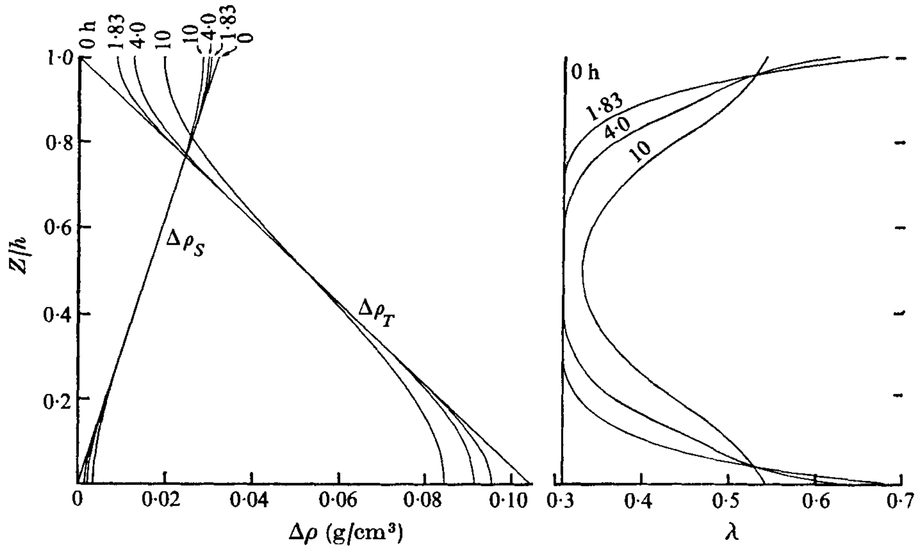

FIGURE 1. Distribution of $\Delta\rho_S$, $\Delta\rho_T$ and λ in a double-diffusive layer with non-diffusive top and bottom boundaries.

were always very small, typically 1 mm in width. When we analysed the diffusion process near the boundaries, it became apparent that this type of experiment could not give us a definitive answer to the question posed.

Consider a layer of fluid with constant, continuous, but opposing gradients of salt and sugar with non-diffusive boundaries top and bottom. Let the diffusivities be 'switched-on' at $t = 0$. The subsequent density changes due to salt $\Delta\rho_T$ and sugar $\Delta\rho_S$ can be calculated using the series solution given by Carslaw & Jaeger (1959) and are shown in figure 1. The vertical co-ordinate is normalized with respect to h, the depth of the layer. In the same figure, the value of $\lambda = \beta(\partial S/\partial z)/\alpha(\partial T/\partial z)$ is also shown. Because of the non-diffusive walls, both $\partial T/\partial z$ and $\partial S/\partial z$ tend to zero at the top and the bottom. Since $\kappa_T > \kappa_S$, $\partial T/\partial z$ approaches zero faster than $\partial S/\partial z$, thus making λ very large in these regions as shown in the figure. It is in these regions that the critical condition is first exceeded and fingers appear. Since these regions are very thin initially, it is conceivable that the fingers occur first with their lateral dimensions of the same order as their vertical dimensions, conforming to the theoretical predictions.

In order to carry out a meaningful experiment, λ in the main body of the layer must be constant for a considerable period to allow the initial small disturbances to grow to finite amplitude so that fingers may be observed. We have come up with an idea which is simple to execute and provides the desired conditions. Consider again a layer of fluid with continuous, constant but opposing gradients of salt and sugar. However, in this case the salt distribution does not tend to zero at the top of the layers; rather it assumes a finite value. Above this two-component layer, there is a layer of fresh water. At the lower boundary, it is assumed that the salt concentration is kept at the saturation value. Again we seek the distributions of $\Delta\rho_T$, $\Delta\rho_S$ and λ as functions of time after the diffusivities were

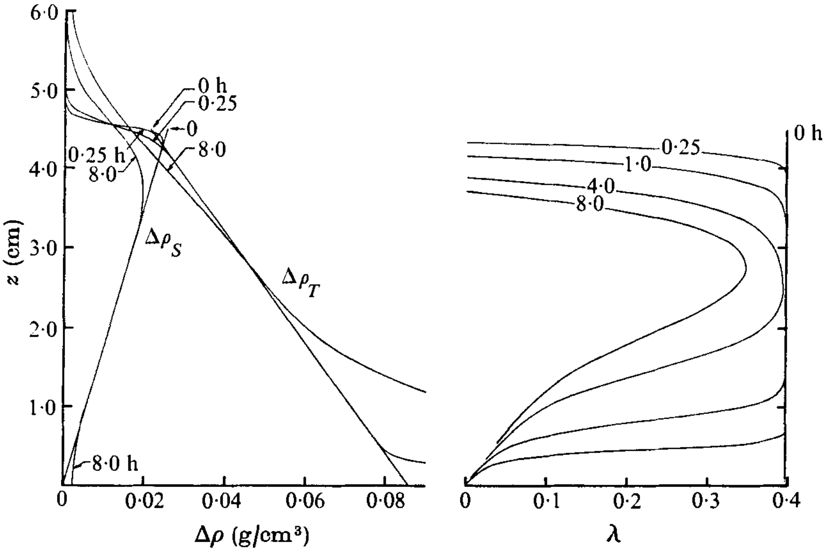

FIGURE 2. Distribution of $\Delta\rho_S$, $\Delta\rho_T$ and λ in a double-diffusive layer with saturated condition for salt solution maintained at the bottom and a layer of fresh water on top.

'switched on' at $t = 0$. Results obtained by numerical integration for the case in which the initial $\lambda = 0{\cdot}398$, the thickness of the double-diffusive layer is 4·5 cm and that of the fresh-water layer is 3·5 cm are shown in figure 2. It can be seen that, up to 1 h, the middle portion of the layer has a constant value of λ equal to the initial value; everywhere else λ is smaller. It is noted that after 8 h diffusion of sugar and salt has not reached the top of the fresh-water layer. This procedure can be carried out easily in the laboratory as described in the next section.

2. Apparatus and procedure

The experiments were carried out in a glass aquarium 40 × 20 × 22 cm deep. In order to provide space for several 'square' cells, the test layer was 4·5 cm in depth. The free surface of the fluid was covered by a piece of Plexiglas to prevent evaporation. A shadowgraph technique was used to visualize both the side and top views of the fingers. The light source was an uncollimated 500 W projection lamp placed about 3 m away from the tank. The top view was obtained by placing tracing paper on the Plexiglas cover and by using two plane mirrors placed at a 45° angle at the bottom and at the top of the tank. A sketch of the experimental arrangement is shown in figure 3.

In order to maintain the bottom boundary condition, the tank was first filled with a layer of saturated salt solution 3 mm thick. Salt crystals were then sprinkled onto the bottom of the tank to ensure that the saturation condition was maintained during the course of the experiment, which usually lasted for about 3 h. The double-diffusive layer was obtained by the two-bucket method of Oster (1965). The inlet fluid was allowed to flow slowly through a fine sponge float in the manner of Turner & Chen (1974) to avoid excessive mixing. It generally required 15 min to obtain a layer 4·5 cm deep. The filling method was calibrated by using

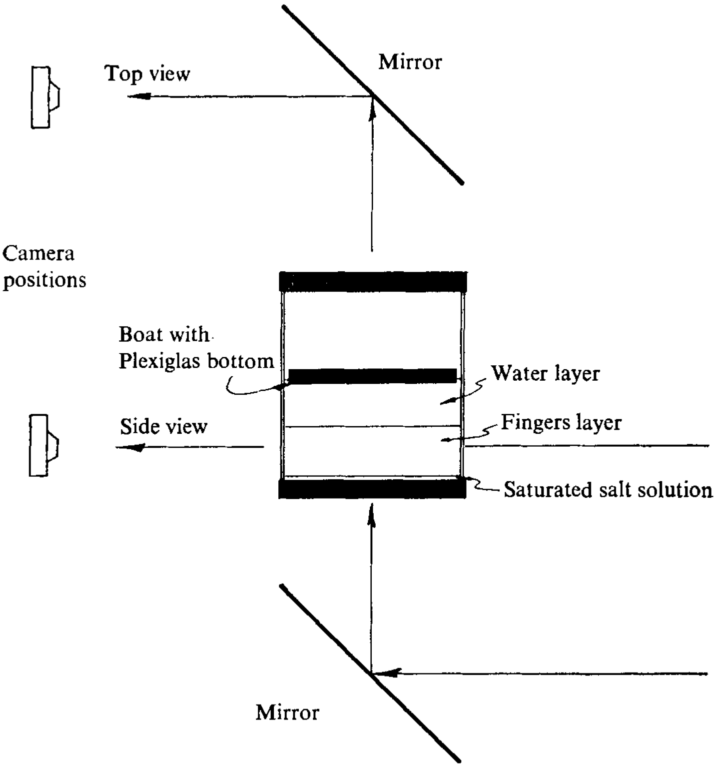

FIGURE 3. Experimental arrangement.

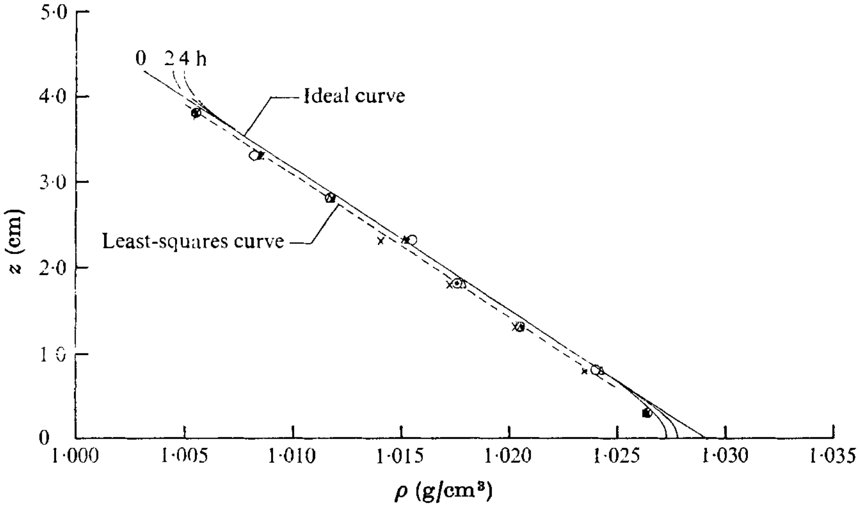

FIGURE 4. Calibration of the two-bucket method. △, ●, ○, ×, probe positions.

Run	Initial λ†	Notes
Subcritical		
64	0·311	Few faint fingers appearing at 1 h. No trace of fingers at 4 h.
65	0·322	No fingers appeared at any time.
Critical		
63	0·333	Faint fingers throughout layer by ½ h. Starting to fade after 2½ h.
66	0·334	Traces remain at 6 h. All gone by 9½ h.
Supercritical		
67	0·347	Fingers stronger than previous cases, appearing throughout layer by ½ h. Starting to fade at 5 h.
68	0·391	Fingers appear throughout in 45 min. Strongest at 1½ h. Start to fade at 2 h. All gone by 3 h.
70	0·398	
69	0·453	Fingers throughout in ½ h. Strongest at 1 h. Starting to fade at 1½ h. All gone by 3 h.

† Values of λ are correct within ± 0·0005.

TABLE 1. Summary of results

fresh water and sugar solution. The vertical density distributions at four different horizontal positions were determined at 5 mm intervals by withdrawing small samples approximately ½ cm^3 each and determining their indices of refraction by a refractometer. The results are shown in figure 4. It is clear from the data that the density distribution was quite uniform horizontally. A least-squares-fit straight line drawn through the data points resulted in a density gradient which is essentially the same as that intended. The entire sampling process, starting with the locations nearest to the free surface, took about 4 h. The set of data nearest to the bottom was the only one which was significantly affected by diffusion.

After the layer depth had reached 4·5 cm, the inlet from the filling bucket was removed. Fresh water was then added through the float to a layer depth of 4–5 cm. It is seen from figure 2 that, with a double-diffusive layer depth of 4·5 cm, the constant-λ layer is initially about 3·5 cm deep. With this depth, the Rayleigh numbers for T and S are of the order of 10^{10}; the constant term in the critical condition can certainly be neglected. The marginal state becomes $R_S/R_T = \tau$, which is approximately $\frac{1}{3}$ for sugar and salt.

3. Results and discussion

A total of eight tests were made and these are summarized in table 1. In all these tests, the maximum density $\rho_0(1+\beta\Delta S)$ of the sugar solution was kept at 1·032 g/cm^3. The maximum density $\rho_0(1-\alpha\Delta T)$ of the salt solution was varied so that the ratio λ assumed values ranging from 0·311 to 0·453. It can be seen from the summary of results that the critical value of λ is $\frac{1}{3}$, the same as the value of τ. In the subcritical case $\lambda = 0{\cdot}311$, the few fingers we observed were probably due to disturbances introduced during filling. As λ becomes supercritical the fingers become stronger in the sense that the density differences between the ascending and descending cells become bigger, thus resulting in greater contrast in the shadowgraph. As λ increases, the transport of sugar and salt through the fingers becomes more vigorous. As a result, the initial gradients of salt and sugar run down faster; the fingers appear to fade at earlier times.

The evolution of the fingering process is best illustrated by a sequence of shadowgraphs taken approximately 15 min apart for the case $\lambda = 0{\cdot}398$. These are shown in figure 5 (plates 1 and 2). The three-digit number appearing beside each photograph denotes the time from the start of the filling process; the first digit denotes hours and the last two digits minutes. Faint fingers appeared at 30 min, and by 45 min the entire layer was filled with fingers. The light band near the bottom was due to refraction of light from the sharp density gradient at the surface of the saturated salt layer. There would have been an alternately dark and light band near the top due to defraction had we not blocked off the light there with a piece of paper.

Starting at 1 h the fingers became stronger and reached their maximum intensity at $1\frac{1}{2}$ h. Owing to the reduced sugar and salt gradient resulting from transport through the fingers, they began to fade at 2 h. By $3\frac{1}{2}$ h no trace of fingers could be observed. From these photographs, it can be seen that the lateral dimensions of the fingers were approximately 1 mm. A top view of the fingers at 1 h 25 min is shown in figure 6 (plate 3). This enlarged photograph shows that the fingers were randomly oriented and that their cross-sections appear to be polygons with 4–6 sides.

It is hoped that our experimental result will stimulate further theoretical investigation of this interesting stability problem.

The financial support of the National Science Foundation for this research under Grant ENG 73–03545 is gratefully acknowledged.

REFERENCES

CARSLAW, H. S. & JAEGER, J. C. 1959 *Conduction of Heat in Solids*, 2nd edn, p. 101. Oxford University Press.

HUPPERT, H. E. & MANINS, P. C. 1973 Limiting conditions for salt-fingering at an interface. *Deep-Sea Res.* **20**, 315–323.

LINDEN, P. F. 1973 On the structure of salt-fingers. *Deep-Sea Res.* **20**, 325–340.

OSTER, G. 1965 Density gradients. *Sci. Am.* **213**, 70–76.

NIELD, D. A. 1967 The thermohaline Rayleigh–Jeffreys problem. *J. Fluid Mech.* **29**, 545–558.

SHIRTCLIFFE, T. G. L. & TURNER, J. S. 1970 Observations of the cell structure of salt fingers. *J. Fluid Mech.* **41**, 707–719.

STERN, M. E. & TURNER, J. S. 1969 Salt fingers and convecting layers. *Deep-Sea Res.* **16**, 497–511.

STERN, M. E. 1960 The 'salt-fountain' and thermohaline convection. *Tellus*, **12**, 172–175.

STOMMEL, H., ARONS, A. B. & BLANCHARD, D. 1956 An oceanographical curiosity: The perpetual salt fountain. *Deep-Sea Res.* **3**, 152–153.

TURNER, J. S. 1967 Salt fingers across a density interface. *Deep-Sea Res.* **14**, 529–611.

TURFER, J. S. & CHEN, C. F. 1974 Two-dimenslonal effects in double-diffusive convection. *J. Fluid Mech.* **63**, 577–592.

WILLIAMS, A. J. 1974 Salt fingers observed in the Mediterranean outflow. *Science*, **185**, 941–943.

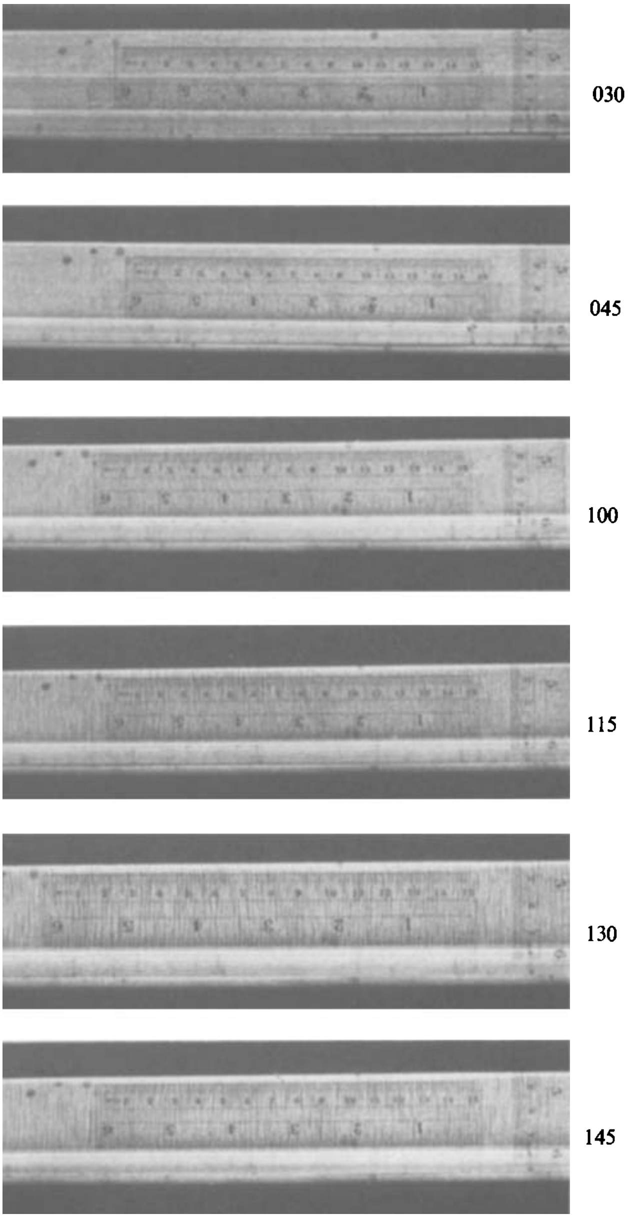

FIGURE 5. Shadowgraphs showing the evolution of salt fingers at $\lambda = 0{\cdot}398$.

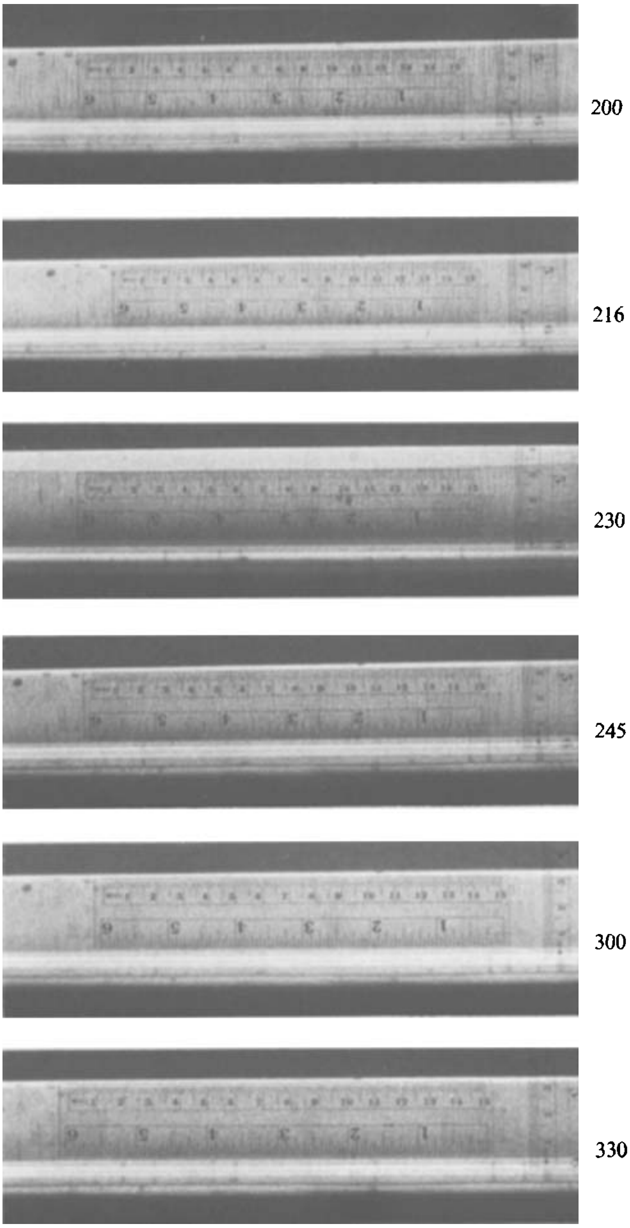

FIGURE 5 (*cont.*) For legend see plate 1.

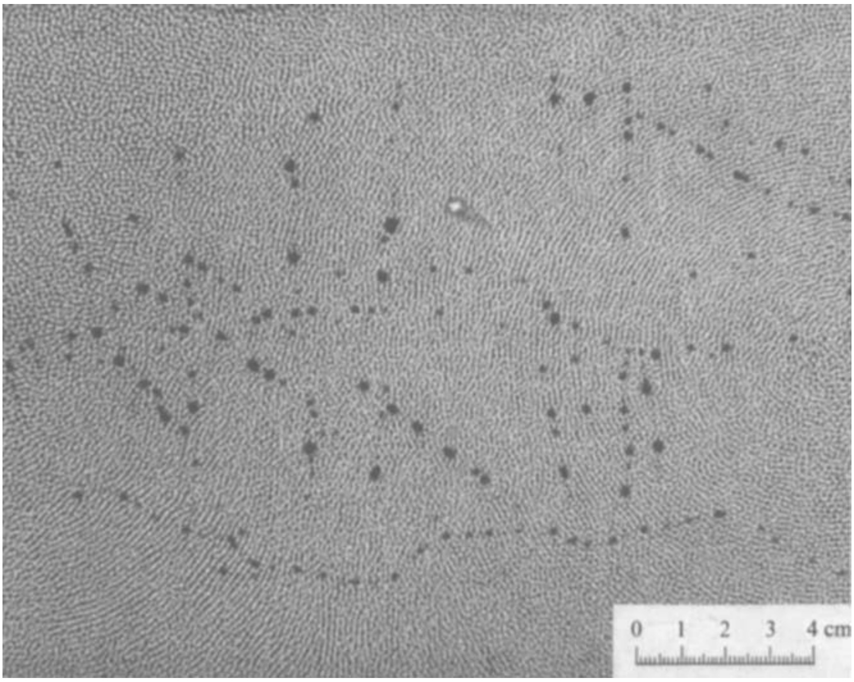

FIGURE 6. Top view of the salt fingers for $\lambda = 0{\cdot}398$ at 1 h 25 min. The black dots are water droplets and the white dot is a drop of glue trapped in the twin-glass bottom.